

# Math Academy

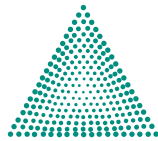
**Let's Go to the Mall!**

Explorations in Combinatorics



**ADVANCING STUDENT ACHIEVEMENT®**

A program of THE ACTUARIAL FOUNDATION



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### A Thank You to Our Corporate Sponsors

The Actuarial Foundation would like to thank the following sponsors for their generosity in funding the production of the Math Academy booklet.

By providing financial support for this project they are assisting The Actuarial Foundation in its pursuit of one of its many aspirations—to provide students with enriched learning opportunities that can help them succeed in their educational and professional endeavors.

The Actuarial Foundation and the companies listed below are collectively ensuring math education remains strong and enjoys a prosperous future.



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# A Letter from the Author



Congratulations! You've just found your answer to the question, "What can I do to create real enthusiasm for mathematics in all my students?"

Created by teachers for teachers, the Math Academy tools and activities included in this booklet were designed to create hands-on activities and a fun learning environment for the teaching of mathematics to our students. On Math Academy days I often found that I couldn't make it from my car to my classroom without being stopped by enthusiastic students wanting to know every detail of the upcoming day's events. Math Academy days contributed to a positive school-wide attitude towards mathematics on our campus.

This booklet contains the *Math Academy – Let's Go to the Mall! Explorations in Combinatorics*, which you can use to enhance your math instruction while staying true to the academic rigor required by the state standards framework.

This effort would not have been possible without support from The Actuarial Foundation. With the help of the Foundation's Advancing Student Achievement (ASA) grant program, which is committed to funding projects that enhance the teaching of mathematics, this Math Academy program came into being.

I sincerely hope that you enjoy implementing the *Let's Go to the Mall! Math Academy* with your students. When you do, you will find that your students engage with mathematics on a whole new level. The Actuarial Foundation is truly a great partner in furthering mathematics education!

*For the kids!*

**Kimberly Rimbey, M.Ed., NBCT (Math)**

Mathematics Specialist

Phoenix, AZ

*If you wish to find out more about my experiences with this and other Math Academies, feel free to contact me via e-mail at [mathacademy@actfnd.org](mailto:mathacademy@actfnd.org).*



# What is a Math Academy?

When the Math Academy concept was first developed, it was designed as a half-day or full-day event which allowed students to deepen their understanding of math while interacting with volunteers from the community (see page 26 for ideas on working with community volunteers). The activities we selected for these events were hands-on, standards-based lessons which applied mathematical principles in real-world scenarios. Each student experienced three to five activities during the course of the event.

Each Math Academy began with a brief school assembly featuring a guest speaker who represented that day's particular theme. Themes included math related to restaurants, sports, nature, shopping, fine arts and other topics, as well as focusing on math-related careers. After the assembly, students rotated to different classrooms where they engaged in various activities related to mathematics and the day's coordinating theme.

Included in this booklet is the *Math Academy – Let's Go to the Mall! Explorations in Combinatorics*, which has all the activities we used for the combinations and permutations Math Academy. This Math Academy is designed to help students understand the connection between various kinds of combinations (multiplication principle and choose numbers) and permutations. You may choose to implement a grade-level or school-wide Math Academy as we originally designed it, or you may prefer to implement these activities in your own classroom. Whichever format you use, keep in mind that the goal is to help your students see the relevance of mathematics in real-life contexts. If you would like more information on the set-up for a school-wide or grade-level Math Academy event, visit The Actuarial Foundation's Web site at [www.actuarialfoundation.org/grant/mathacademy.html](http://www.actuarialfoundation.org/grant/mathacademy.html)



# Getting Started

## Math Academy Format

You may choose to conduct your Math Academy as a school-wide event, as a grade-level rotation, or as a single-classroom experience. If you will be holding your Math Academy for a single classroom, you may want to invite a guest speaker to speak with your class about how mathematics is used in his/her job (rather than putting on a school-wide or grade-level assembly as described below).

## Math Academy Schedule

*Schedule and times may vary depending on format being used.*

- Opening assembly (optional) — 15 minutes
- Directions and Math Journals — 15 minutes
- Activity Rotations — 30–45 minutes per activity
- Assessment and Closure — 15 minutes

## Opening Assembly/Directions

To build enthusiasm and to focus attention, have everyone participate in the *Math Academy Chant* (for younger students):

“You and me, we all agree —  
Math is fun as you will see!  
It makes us think, it makes us strong,  
It helps us learn even when we’re wrong.  
You and me, we all agree —  
Math is fun at the Math Academy!”

### Introduction

Announce to the students that today they will be working on projects dealing with combinations and permutations.

### Guest Speaker Presentation (optional)

Beforehand, arrange for someone in the sales industry to talk to the students about the mathematics in the mall (they may or may not connect to combinations and permutations, but encourage them to do so). Possible guest speakers may include someone from the mall, ice cream shop, clothing shop, or any retail store. Although this guest speaker will primarily be focused on the shopping side of this Math Academy theme, you may want to work with him/her ahead of time to assist with tying in mathematical ideas to the talk.

## Use of Math Journals

Students record their findings in their “Math Journals” during each activity. These journals should contain all recording sheets for the activities as well as blank paper for the extension activities. Before beginning the Activities Rotation, students should spend about 10 minutes writing in their math journals, including their reflections from the assembly as well as briefly describing what they already know about combinations and/or permutations.



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## Theme

Combinations and Permutations —  
Using ideas from the mall to facilitate discussions focused on combinatorics.

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## Objective

Students will build, describe, and use numerical operations to determine combinations and permutations.

# Getting Started (continued)

## ► TEACHER TIPS

- We asked our students to use graphic organizers to record their thoughts and findings in their math journals. This creates a great connection to our language arts curriculum.
- The best results are achieved when 45 minutes are allowed in each classroom – about 30 minutes for the activity and 15 minutes for clean-up and reflection. The students can use their math journals to record their findings during the activities and write reflections on the journal's blank pages.
- Following the combinations and permutations activities in this booklet, you will find a student quiz as well as sample surveys for students and teachers so that you can evaluate the success of your Math Academy. The Actuarial Foundation would love to hear all about your success and will send a \$50 Gift Certificate to the first 100 teachers who implement these activities and submit the results. For more information and to download the evaluation form, go to [www.actuarialfoundation.org/grant/mathacademy.html](http://www.actuarialfoundation.org/grant/mathacademy.html).

## Activities Rotation

If multiple classes are participating in the Math Academy, each classroom should host a different activity so students will rotate from classroom to classroom in order to complete each activity. If only one class is participating, the students may rotate from one activity to the next around the room, or they may do each activity as a whole class, one activity after the other. Activities begin on page 7. For best results, plan on three to five activities for your Math Academy.

## Closure and Assessment

Once all activities are completed, the students may return to their homeroom classes for final reflections and assessment. See pages 22 and 24 or sample quiz and survey.

## Key Vocabulary

- Combinatorics
- Combinations
- Permutations
- Multiplication Principle
- Choose Numbers
- Factorials

## ► TEACHER TIPS: CUSTOMIZING THESE ACTIVITIES

- Use the “Procedures” section in each activity as a skeleton for the lesson.
- Look through the “Suggestions for Customizing This Activity” section.
- Select the suggestion(s) which best fits with the combinatorics concepts on which you want your students to focus.

# Math Activity 1: *It's a Wrap with Combinations*

**Objective** The students will determine combinations using the “multiplication principle” and then model the results using a tree diagram.

**Materials**

- Pencils
- Clear bags for paper manipulatives
- Paper manipulatives copied, cut and placed in plastic bags (see page 8)
- “It’s a Wrap” combinations sheet for each student (see page 9)

**Procedures**

1. **Optional:** Set the stage for the wrapping paper store – put a sign on the outside of your door, decorate your room, wear an apron, make a “choices” sign, etc.
2. Prepare the paper manipulatives and put in clear bags.  
NOTE: Be sure to include more of each item than is needed so that students must think about when to stop putting combinations together.
3. Discuss the problem with the students (see “It’s a Wrap” combinations sheet).
4. Demonstrate how to make wrapping paper/bow combinations using exactly one paper and one bow.
5. Have students work in groups to create all of the possible wraps using exactly one paper and one bow.
6. Once groups are finished, demonstrate how to represent all of the combinations using a tree diagram.
7. Have students create their own tree diagrams to match their wrapping creations.
8. Closure – discuss why there are exactly nine possible wrap combinations using only one paper and one bow.

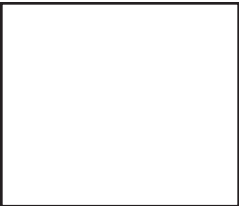
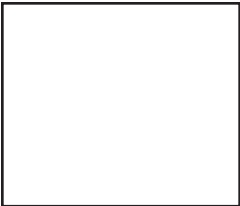
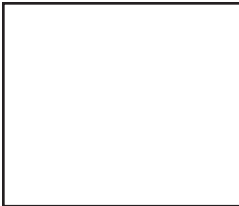
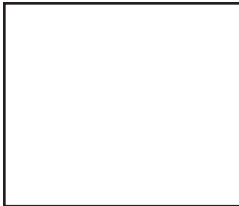
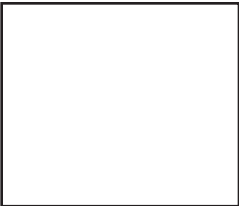
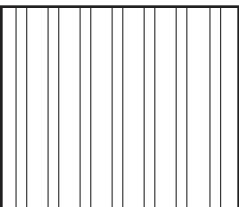
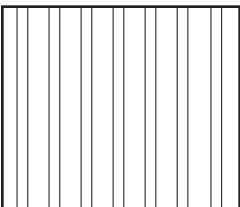
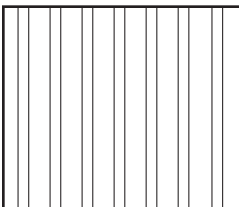
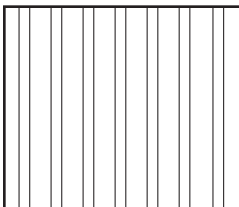
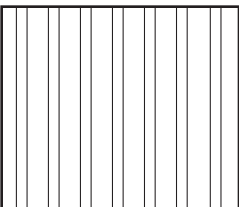
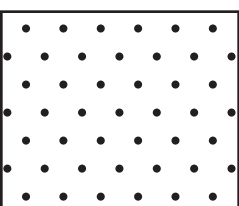
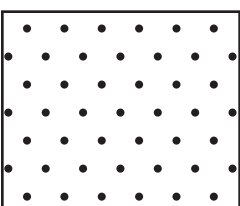
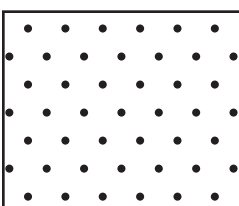
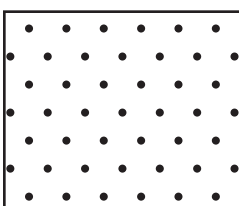
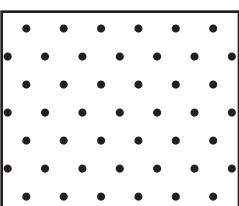














**Suggestions for Customizing This Activity**

- This problem is set up to include three kinds of paper and three kinds of bows (9 possible combinations). Put in more papers and/or more bows to add an extra challenge.
- Add a third set of objects (in this case, ribbons or stickers).
- Discuss the multiplication principle (determining the number of combinations by multiplying the number of items in each set) and have students relate it directly to their diagrams and/or pictures (in this case,  $3 \times 3 = 9$  possible combinations).
- Have students create an organized list, table, or array model to illustrate their combinations rather than a tree diagram.



# Math Activity 1: *It's a Wrap Cut-Outs*

Copy the “wrapping paper” and “bows” below. Cut apart to be used with “It’s a Wrap.”

				
Solid	Solid	Solid	Solid	Solid
				
Stripes	Stripes	Stripes	Stripes	Stripes
				
Polka Dots	Polka Dots	Polka Dots	Polka Dots	Polka Dots
				
Simple Bow	Simple Bow	Simple Bow	Simple Bow	Simple Bow
				
Curly Ribbon	Curly Ribbon	Curly Ribbon	Curly Ribbon	Curly Ribbon
				
Round Bow	Round Bow	Round Bow	Round Bow	Round Bow



# Math Activity 1: *It's a Wrap Combination Sheet*

Maria is getting her mother's gift wrapped at the mall. The sign below shows that the price range she is looking at has three wrapping paper choices and three bow choices:

Wrapping Paper Choices	Bow Choices
Solid	Simple Bow
Stripes	Curly Ribbon
Polka Dots	Round Bow

What are all the different combinations that can be made using exactly one wrapping paper and one bow?

Make a tree diagram below to show all the possible combinations. Use words, numbers, and pictures to demonstrate/explain how you arrived at your answer.

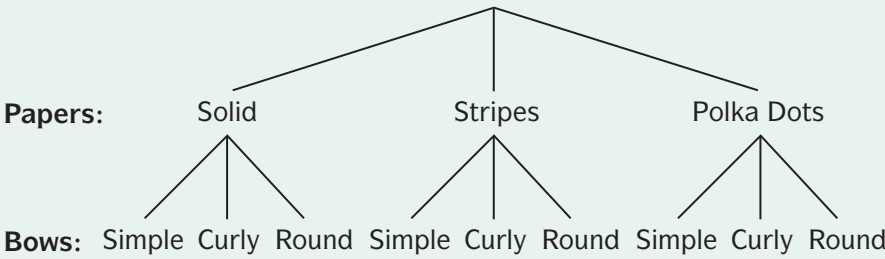
# Math Activity 1: *It's a Wrap with Combinations*



## Teachers' Answer Key

### Mathematical Patterns in this problem:

- If there are 3 papers and 3 bows, there are 9 possible combinations using exactly one paper and one bow.
- **Tree Diagram:**



- **Note:** You may choose to have the students use the following models as well.
- **Organized List/Table**

Paper	Bow
Solid	Simple
Solid	Curly
Solid	Round
Stripes	Simple
Stripes	Curly
Stripes	Round
Polka Dots	Simple
Polka Dots	Curly
Polka Dots	Round

- **Array**

		BOWS		
		Simple (S)	Curly (C)	Round (R)
PAPERS	Polka Dots (P)	P-S	P-C	P-R
	Stripes (St)	St-S	St-C	St-R
	Solid (S)	S-S	S-C	S-R

- **Multiplication Principle:**

$3 \times 3 = 9$

## Math Activity 2: *Be a Sport with Combinations*

### Objective

The students will determine combinations using “choose numbers” while discovering, identifying, recording, and verbalizing the arithmetic patterns for various choose-number combinations. The students will model their results as an organized list on a chart.

### Materials

- Pencils
- Clear bags for paper manipulatives
- Paper manipulatives copied, cut and placed in plastic bags (see page 13)
- “The Big Sale” combinations sheet for each student (see page 14)

### Procedures

1. **Optional:** Set the stage for the “Be a Sport” store – put a sign on the outside of your door, decorate your room, and/or dress up like someone working at a sports apparel store.
2. Copy and cut apart the paper manipulatives and put in clear bags. **Note:** Be sure to include more of each item than is needed so that students must think about when to stop putting combinations together.
3. Discuss the problem with the students (see “Be a Sport” combinations sheet). Explain that the store is having a 2-item special. Each customer may select exactly two items (must be different) and put them in a bag. Their job is to discover how many different bags can be made when selecting two items from six choices.
4. Demonstrate how to select exactly two items from the six choices. Emphasize that the two items may never be identical (e.g.: they cannot select two baseball hats at the same time).
5. Have students work in groups to create all of the possible 2-item specials by selecting exactly two unique items for each bag.
6. Once groups are finished, demonstrate how to represent all of the combinations using a chart or organized list.
7. Have students create their own charts and/or organized lists to demonstrate their results.
8. Closure – discuss why there are exactly 15 possible combinations.



## Math Activity 2: *Be a Sport with Combinations*

### ► TEACHER TIPS

#### The Famous “Handshake Problem”

This widely-used scenario has been used in mathematics classes for many years. Ask the student: If a group of people got together and each person shook the hand of every other person in the group exactly one time, how many handshakes would take took place? The table below shows the answers:

# of people in group	# of handshakes
2	1 (1)
3	3 (2+1)
4	6 (3+2+1)
5	10 (4+3+2+1)
6	15 (5+4+3+2+1)

This is another example of “choose numbers”. In this case, no matter how many people are in the group, two people are “chosen” at a time to shake hands. You may want to have the students act this out.

### Ideas for Customizing This Activity

- Have students create a tree diagram to illustrate their combinations rather than an organized list.
- Change the number of items being used and/or the number of items being chosen (start with 6 items and choose 1, 2, 3, 4, 5, or 6; start with 5 items and choose 1, 2, 3, 4, or 5; start with 4 items and choose 1, 2, 3, or 4; start with 3 items and choose 1, 2, or 3; start with 2 items and choose 1 or 2; start with 1 item and choose 1).
- Ask students to create a chart or diagram to show the patterns for each of the combinations created in the above option.
- Discuss how “choose number” combinations differ from the “multiplication principle” combinations (lessons one in this booklet).
- Ask students to look for patterns when organizing their combinations.
- Ask students to derive an algorithm for determining the number of combinations possible.
- Change the number of items available.
- **Advanced:** Develop the formula for finding choose numbers. The formula for finding choose numbers is rarely taught in middle school. The beauty of this problem for middle school students is in allowing the students the opportunity to visualize, identify, and verbalize patterns. However, if you choose to introduce the formula, you will want to be sure they are already very familiar with factorials.

If  $n$  = number of items available and  $t$  = how many items chosen, then the formula will be as follows:

$$\frac{n!}{(t!)(n-t)!}$$



## Math Activity 2: *Be a Sport Cut-Outs*



# Math Activity 2: *The Big Sale Combination Sheet*

Be A Sport

SALE!

Jersey  
Baseball cap  
Shorts  
T-shirt  
Running shoes  
Sweat pants

The sports apparel store at the mall, Be a Sport, is having a great sale this weekend! Each customer may choose exactly two items from the list and purchase them both for a great price. The trick is that each 2-item special must have two different items (for example, they may not purchase two T-shirts at the same time).

What are all the different combinations that can be made by choosing exactly two items?

Use the paper items to discover how many combinations are possible (remember, you may not put two of the same item together). Then make an organized list in the chart below to show all the possible combinations. Use words, numbers, and pictures to demonstrate/explain how you arrived at your answer.

Item 1	Item 2

# Math Activity 2: *Be a Sport with Combinations*

## Teachers' Answer Key

Chart/organized list for choosing two items from six choices (6 choose 2):

- **Note to teachers:** order will vary, but look for patterns

First Item	Second Item
Jersey	Baseball cap
Jersey	Shorts
Jersey	T-shirt
Jersey	Running shoes
Jersey	Sweat pants
Baseball cap	Shorts
Baseball cap	T-shirt
Baseball cap	Running shoes
Baseball cap	Sweat pants
Shorts	T-shirt
Shorts	Running shoes
Shorts	Sweat pants
T-shirt	Running shoes
T-shirt	Sweat pants
Running shoes	Sweat pants



**Mathematical Patterns in this problem:**

- **Choose 2:** When there are 4 items and they are choosing 2, there is a  $3 + 2 + 1$  pattern; When there are 5 items and they are choosing 2, there is a  $4 + 3 + 2 + 1$  pattern; When there are 6 items and they are choosing 2, there is a  $5 + 4 + 3 + 2 + 1$  pattern; etc.:

<b>4 choose 2</b>	$3 + 2 + 1 = 6$ combinations
<b>5 choose 2</b>	$4 + 3 + 2 + 1 = 10$ combinations
<b>6 choose 2</b>	$5 + 4 + 3 + 2 + 1 = 15$ combinations
<b>7 choose 2</b>	$6 + 5 + 4 + 3 + 2 + 1 = 21$ combinations
<b>8 choose 2</b>	$7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$ combinations

- **Choose 3:** When there are 4 items and they are choosing 3, there is a  $2 + 1$  pattern; When there are 5 items and they are choosing 3, there is a  $3 + 2 + 1$  pattern; When there are 6 items and they are choosing 3, there is a  $4 + 3 + 2 + 1$  pattern; etc.:

<b>4 choose 3</b>	$2 + 1 = 3$ combinations
<b>5 choose 3</b>	$3 + 2 + 1 = 6$ combinations
<b>6 choose 3</b>	$4 + 3 + 2 + 1 = 10$ combinations
<b>7 choose 3</b>	$5 + 4 + 3 + 2 + 1 = 15$ combinations
<b>8 choose 3</b>	$6 + 5 + 4 + 3 + 2 + 1 = 21$ combinations

## Math Activity 2: *Be a Sport with Combinations*



### Teachers' Answer Key (continued)

- Choosing different amounts from the same original number of items:

4 choose 1	4 combinations
4 choose 2	$3 + 2 + 1 = 6$ combinations
4 choose 3	$2 + 1 = 3$ combinations
4 choose 4	1 combination

5 choose 1	5 combinations
5 choose 2	$4 + 3 + 2 + 1 = 10$ combinations
5 choose 3	$3 + 2 + 1 = 6$ combinations
5 choose 4	$2 + 1 = 3$ combinations
5 choose 5	1 combination

6 choose 1	6 combinations
6 choose 2	$5 + 4 + 3 + 2 + 1 = 15$ combinations
6 choose 3	$4 + 3 + 2 + 1 = 10$ combinations
6 choose 4	$3 + 2 + 1 = 6$ combinations
6 choose 5	$2 + 1 = 3$ combinations
6 choose 6	1 combination

**Using the formula:** The formula for finding 6 choose two will include factorial for the number of items available, the factorial for the number of items chosen, and the factorial for the number of items NOT chosen. It will look like this:

$$\frac{6!}{(2!)(6-2)!}$$

► REMINDER:

$$\frac{n!}{(t!)(n-t)!}$$



# Math Activity 3: *Ice Cream Cone Permutations*

## Objective

The students will determine permutations while discovering, identifying, recording, and verbalizing the arithmetic patterns for various amounts.

## Materials

- Paper ice cream scoops (3 to 6 flavors cut from construction paper)
- Blank Paper
- Pencils
- Paper triangles to represent cones onto which to place the paper scoops
- Colored pencils, crayons, or markers

## Procedures

1. **Optional:** Set the stage for the ice cream shop – you may want to put a sign on the outside of your door to let everyone know they are going into the Ice Cream Shop. If you wish, you can even decorate your room and/or wear apparel appropriate for an ice cream shop worker.
2. Discuss the problem with the kids.
  - Today everyone will get a 3-scoop ice cream cone.
  - The big decision is this: in what order do you want your scoops arranged on your cone?
  - Use the paper ice cream to discover all of the possible orders your three scoops can be arranged.
  - Organize the paper ice cream cones so that patterns can be easily detected.
  - Record all of the possible *permutations* (order in which they can be arranged) in an organized fashion using a tree diagram, organized list, table, picture, etc.
3. Have students work in groups or individually to use the paper scoops and cones to find all of the orders in which three scoops can be arranged.
4. Have students record their results as a tree diagram, organized list, chart, etc., on the blank paper. It's important that the students see a connection between the manipulatives and the diagrams they create.
5. Discuss the mathematical patterns that exist in this problem (see teacher answer key for examples).
6. **Just For Fun:** Provide real ice cream for a 2 or 3-scoop special with your class following the activity.



# Math Activity 3: *Ice Cream Cone Permutations*

### Ideas for Customizing This Activity

- Introduce the concept of factorials – *Anno’s Mysterious Multiplying Jar* by Mitsumasa Anno is a great literature selection which explains factorials using several models
- Repeat this activity using 1 scoop, 2 scoops, 4 scoops, etc. Place the results on a chart in order to detect arithmetic patterns (see Teachers Answer Key for examples).
- Repeat this activity using a different context in which students may order items into different arrangements. Possibilities may include:
  - children standing in line (e.g., In how many different orders can Anna, Brock, Char, and Dave stand in line at the Ice Cream Store?)
  - placing digits in different orders (e.g., How many different 3-digit numbers can you make given the digits 7, 8, and 9?)

### Anno’s Mysterious Multiplying Jar by Mitsumasa Anno

This amazing book demonstrates the concept of factorials in such a concrete way! It starts with a jar, and inside the jar is one island. On that island are two countries. In each country are three mountains, etc. Each page shows the multiplicative growth in pictures and numbers, all the way to 10!

### Teachers’ Answer Key

Mathematical Patterns in this problem:

1 scoop	$1! = 1$ permutation
2 scoops	$2! = 2 \times 1 = 2$ permutations
3 scoops	$3! = 3 \times 2 \times 1 = 6$ permutations
4 scoops	$4! = 4 \times 3 \times 2 \times 1 = 24$ permutations
5 scoops	$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ permutations
6 scoops	$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ permutations

# Math Activity 4: *Create-a-Bear Permutations*

## Objective

The students will determine permutations while discovering, identifying, recording, and verbalizing the arithmetic patterns for various amounts.

## Materials

- Blank Paper
- Pencils
- Colored pencils, crayons, or markers
- Clear bags for paper manipulatives
- Paper manipulatives copied, cut and placed in plastic bags (see page 17)

## Procedures

1. Set the stage for the students as if they are working at the “Create-a-Bear” store in the mall. Their job is to make a poster showing all of the possible ways the bears can be placed in a line (permutations).
2. Discuss the problem with the kids.
  - Today the students will be working at the Create-a-Bear store in the mall. They will be creating a display to show all of the little children how they can dress the bears.
  - The big decision is this: in what order do you want to line the bears up in the display?
  - Use the paper bears to discover all of the possible orders in which you can put them in line. Suggestion: use three bears to begin with, then more or less.
  - Organize the different arrangements so that patterns can be easily detected using a chart, organized list, or other model.
  - Record all of the possible permutations (order in which they can be arranged) in an organized fashion using a tree diagram, organized list, table, picture, etc.
3. Have students work in groups or individually to use the paper bears to find all of the orders in which the bears can be arranged in line if there are 3 bears. (You may change the number as desired.)
4. Have students record their results as a tree diagram, organized list, chart, etc., on the blank paper. It’s important that the students see a connection between the manipulatives and the diagrams they create.
5. Discuss the mathematical patterns that exist in this problem (see teacher answer key for examples).



## Math Activity 4: *Create-a-Bear Permutations*



### Ideas for Customizing This Activity

- Introduce the concept of factorials – *Anno's Mysterious Multiplying Jar* by Mitsumasa Anno is a great literature selection which explains factorials using several models (see page 18 for a description).
- Repeat this activity using 1 bear, 2 bears, 4 bears, etc. Place the results on a chart in order to detect arithmetic patterns (see Teachers Answer Key for examples).
- Repeat this activity using a different context in which students may order items into different arrangements. Possibilities may include
  - children standing in line (e.g., In how many different orders can Anna, Brock, Char, and Dave stand in line at the Create a Bear Store?)
  - placing digits in different orders (e.g., How many different 3-digit numbers can you make given the digits 7, 8, and 9?)
- See Ice Cream Cone Permutations in this booklet.

### Teachers' Answer Key

Mathematical Patterns in this problem:

1 bear	$1! = 1$ permutation (order in which to line up the bear)
2 bears	$2! = 2 \times 1 = 2$ permutations (orders in which to line up the bears)
3 bears	$3! = 3 \times 2 \times 1 = 6$ permutations (orders in which to line up the bears)
4 bears	$4! = 4 \times 3 \times 2 \times 1 = 24$ permutations (orders in which to line up the bears)
5 bears	$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ permutations (orders in which to line up the bears)
6 bears	$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ permutations (orders in which to line up the bears)



## Math Activity 4: *Create-a-Bear Permutations*

Copy the “bears” below. Cut apart to be used with “Create-a-Bear Permutations”



Green



Green



Green



Green



Green



Green



Green



Green



Green



Green



Red



Red



Red



Red



Red



Red



Red



Red



Red



Red



Blue



Blue



Blue



Blue



Blue



Blue



Blue



Blue



Blue



Blue

# Sample Quiz

Name \_\_\_\_\_

1. Draw a tree diagram which shows the possible outfits which can be created from 3 pairs of shorts and 4 shirts. How many combinations were possible?
2. The pet store at the mall is offering a special on food and water bowls. There are 9 different food bowls and 6 different water bowls from which to choose. If each set must contain exactly one food bowl and one water bowl, how many different combinations are possible? (Show your work.)
3. The stir-fry restaurant at the mall is offering a special on 2-item stir-fries. If there are 5 items to choose from, how many 2-item combinations are possible? (Show your work.)

# Sample Quiz *(continued)*

4. The jewelry store at the mall just put 7 items in the sale bin. They are selling them in sets of three. How many combinations of three different items can be put together (no duplicates in any set)? (Show your work.)
5. Four students are standing in line to go to the movies at the mall. In how many different orders can they stand in line? (Show your work.)
6. The toy store at the mall is setting up a new display. There are six different toys they are putting in the window. In how many orders can the toys be placed? (Show your work.)

# Sample Student Survey

1. What job would you like to have when you grow up?

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2. What kind of math skills do you think you will need to do that job?

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3. Rank each of these statements on a scale of 1 to 5 (1 being lowest rank):

• I like math in school.	1	2	3	4	5
• I use math outside of school.	1	2	3	4	5
• The math I learn at school is helpful.	1	2	3	4	5
• I am good at math.	1	2	3	4	5
• I liked participating in the <i>Let's Go to the Mall!</i> Math Academy	1	2	3	4	5
• I learned a lot about math during the Math Academy.	1	2	3	4	5
• I would like to participate in Math Academies again in the future.	1	2	3	4	5

4. The best thing about the Math Academy was:

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5. The worst thing about the Math Academy was:

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6. Do you know what an actuary is?

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# Sample Teacher Survey

1. What format did you use for this Math Academy?  
☐ classroom      ☐ all classrooms in grade level      ☐ school-wide to all grade levels
2. What grade-level(s) used this Math Academy lesson? \_\_\_\_\_
3. Which Math Academy activities did you utilize with your students?    1    2    3    4    5    6
4. For the following two questions, please use a ranking scale of 1 through 5  
(5 = great; 3 = mediocre; 1 = poor).  
\_\_\_\_\_ Overall, how would you rank this Math Academy?  
\_\_\_\_\_ How would you rank the activities you presented/taught?
5. Would you recommend these activities be used again?  
☐ Yes      ☐ No  
Comments: \_\_\_\_\_  
\_\_\_\_\_
6. Do you think your students now have a better understanding of combinations and permutations?  
☐ Yes      ☐ No  
Comments: \_\_\_\_\_  
\_\_\_\_\_
7. Would you like to participate in another Math Academy?  
☐ Yes      ☐ No  
Comments: \_\_\_\_\_  
\_\_\_\_\_
8. Please let us know how well your Math Academy went.  
\_\_\_\_\_  
\_\_\_\_\_
9. Comments, ideas, suggestions:  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

# Community Involvement

One of the most beneficial aspects of our Math Academy program is the actuarial mentors/volunteers who interact with our students during each Math Academy activity. Our actuarial mentors take time away from their usual work responsibilities on the days of our events so they can help out in the classrooms. Please take the time to contact The Actuarial Foundation at [mathacademy@actfnd.org](mailto:mathacademy@actfnd.org) to find out if actuaries are available in your area.

If you do not have actuaries available in your community, you may want to consider requesting the assistance of parents, community members, or business partners who would be willing to work with the students during the activities.

After securing your mentors/volunteers, it is critical to identify a Lead Mentor who will serve as the liaison between you and the other mentors. The Lead should e-mail all communications from the teachers to the mentors, set up schedules, send reminders before each event, etc.

## Mentor Training Session

- Distribute the Math Academy schedule.
- Distribute copies of the Math Academy activities.
- **Pedagogy** – Discuss some simple teaching techniques.
- **Management** – Assure mentors that the teachers will handle all discipline. Discuss preventative management techniques such as proximity and having activities well-prepared to avoid student down-time.
- **Brainstorm** – Allow some time during your training to take any ideas or suggestions from your mentors. Allow time for questions and answers.
- **Assigning mentors** – Assigning mentors to the same classroom throughout the year will help build stronger relationships with the students and teachers.
- **E-mail exchange** – Collect everyone's e-mail addresses for easy communication between mentors and teachers.
- **School tour** – End your training with a school tour. Be sure your mentors know the key locations of your school including the sign-in book (and procedures), restrooms, principal's office, and classrooms. If possible, include a map in their take-home materials so they can find their teachers' classrooms once they receive their assignments.

## WHAT IS AN ACTUARY?

An *actuary* is an expert who deals with numbers and percentages, also known as statistics. Actuaries provide advice to businesses, governments, and organizations to help answer questions about what to expect in planning for the future.

To find out more about the actuarial profession visit **BeAnActuary.org**.

## ► TEACHER TIPS

- The number of volunteer mentors you have will depend upon the format and number of students involved in the program. Although you don't need volunteer mentors at the classroom level, students find these volunteer mentors to be fun.

# Alignment with Standards

The unit included in this *Let's Go to the Mall!* booklet draws from the field of discrete mathematics. According to the NCTM *Principles and Standards*, “the main topics of discrete mathematics are distributed across the Standards instead of receiving separate treatment, and they span the years from pre-kindergarten through grade 12. As an active branch of contemporary mathematics that is widely used in business and industry, discrete mathematics should be an integral part of the school mathematics curriculum, and these topics naturally occur throughout the other strands of mathematics.”

Although the content for combinatorics is integrated throughout the NCTM content standards, the following pK-12 process standards are specifically addressed in this booklet:

## Problem Solving:

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

## Connections:

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

## Representation:

- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena

*You will want to check with your own state framework to select performance objectives which are specific to your students.*



[illegible]

- ☐ Determine the date, time, and schedule for your Math Academy
- ☐ Identify the objectives to be reinforced through this Math Academy
- ☐ Plan the opening assembly, if applicable
- ☐ Confirm the schedule and content with guest speaker, if applicable
- ☐ Customize the activities enclosed in this booklet
- ☐ Make copies of the activities, quiz and surveys
- ☐ Purchase and/or gather materials

**Note:** Most hands-on materials for these activities can be created by copying the black-line masters throughout the booklet. In addition, you will need the following items:

- ☐ small, clear bags to organize paper manipulatives
- ☐ colored paper
- ☐ colored pencils, markers, and/or crayons

- ☐ Make math journals for all students
- ☐ Distribute materials to other participating teachers, if applicable

# More Resources from The Actuarial Foundation



## ADVANCING STUDENT ACHIEVEMENT.

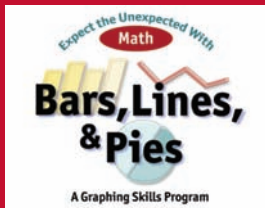
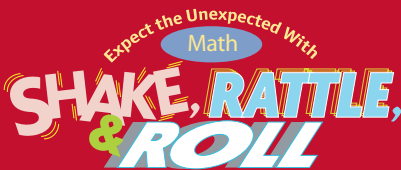
A program of THE ACTUARIAL FOUNDATION

### Advancing Student Achievement Grants

Advancing Student Achievement helps support your efforts in the classroom by integrating hands-on, practical mathematics skills brought to life by practicing professionals into your everyday curriculum.

As part of that program, The Actuarial Foundation offers funding of mentoring programs that involve actuaries in supporting your school's teaching of mathematics.

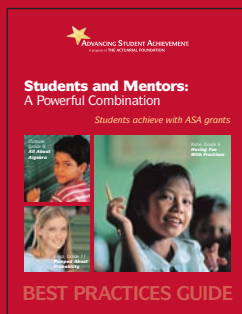
For information on this program and to find out how you can apply for an ASA grant, visit [www.actuarialfoundation.org/grant/](http://www.actuarialfoundation.org/grant/).



### Expect the Unexpected with Math

*Shake, Rattle, & Roll* and *Bars, Lines, & Pies*, educational programs funded by the Foundation and developed and distributed by Scholastic, the global children's publishing education and media company, are designed to provide teachers and students with math literacy-based materials that meet national standards and are in alignment with core school curriculum. These skill-building programs provide lesson plans, activities and other teaching resources while incorporating and applying actuaries' natural mathematics expertise in real world situations.

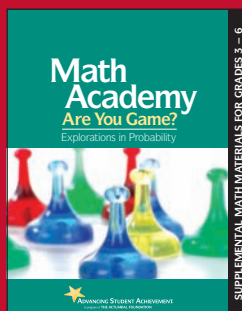
To learn more about the program, or to download a copy, visit [www.actuarialfoundation.org/grant/index.html](http://www.actuarialfoundation.org/grant/index.html).



### Best Practices Guide

This guide features a compilation of research on the value of mentoring, combined with 15 case histories of programs funded by the Foundation, each of which includes information on program design and results.

To request a hard copy of the Best Practices Guide, send an e-mail to [asa@actfnd.org](mailto:asa@actfnd.org) or to download a copy, visit [www.actuarialfoundation.org/grant/bestpractices.html](http://www.actuarialfoundation.org/grant/bestpractices.html).



### Math Academy Series

Real-life applications of mathematics while having fun? Is it possible? Yes! Created by teachers for teachers, the Math Academy tools and activities included in these booklets were designed to create hands-on activities and a fun learning environment for the teaching of mathematics to students. You can use Math Academy activities to enhance your math instruction while staying true to the academic rigor required by the state standards framework. To download other booklets visit: [www.actuarialfoundation.org/youth/mathacademy.html](http://www.actuarialfoundation.org/youth/mathacademy.html).





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